

**Problem set 71**

- Find the arc length of the largest rectangle which can be inscribed in the region bounded by the  $x$  axis and the graph of  $y = 12 - x^2$ . (Begin by drawing the rectangle. Let  $x$  be the distance from the origin to the lower right-hand corner of the rectangle. Express the area of the rectangle in terms of  $x$ .)
- A steady force of 20 newtons is applied to an object to move it 30 meters. What is the work done by the force?
- A variable force  $F(x) = \frac{1}{2}x^3 + x$  newtons is applied to an object to move it from  $x = 0$  to  $x = 3$  meters. Find the work done by the force.
- The spring constant for a spring is 3 newtons per meter. How much work is done in stretching the spring from 2 to 4 meters?
- Katherine and Martine estimated that the velocity of an object moving along the  $x$  axis at a time  $t$  was given by

$$v(t) = 3t + 4$$

- Find the distance the object moved between  $t = 1$  and  $t = 4$ .
- Suppose  $f(x) = ax^3 + bx$ . Find  $a$  and  $b$  if the graph of  $f$  passes through  $(1, -1)$  and  $f'(1) = 3$ .
- Suppose  $f$  is a function whose slope at any point is twice its  $x$  coordinate. If the graph of  $f$  passes through  $(1, 1)$ , find the equation of  $f$ .

Integrate:

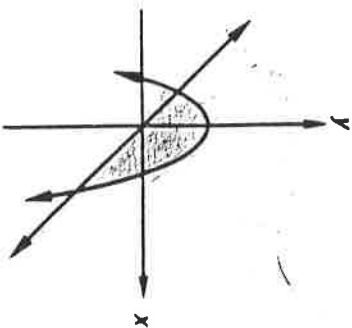
$$8. \int \cos\left(2x - \frac{\pi}{2}\right) dx$$

$$9. \int \frac{x^{-1}}{\sqrt{x}} dx$$

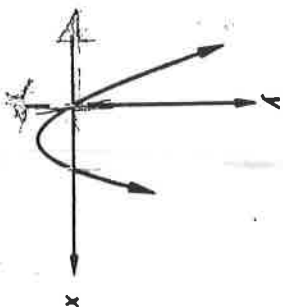
$$10. \int \cos t \sqrt{\sin t} dt$$

$$11. \int \tan^3 x \sec^2 x dx$$

- Find the area of the region enclosed by the graphs of  $y = 2 - x^2$  and  $y = -x$ .



- Find the area between the graph of  $y = x(x - 2)$  and the  $x$  axis between  $x = -1$  and  $x = 2$ .



- Suppose  $y = f(x) = x^3 + x$ . Write an implicit equation of  $f^{-1}$ .
- A ball is thrown straight up from ground level. Its height above the ground in feet at time  $t$  is given by

$$h(t) = 200t - 16t^2$$

How high will the ball go?

- Differentiate:  $y = \frac{e^{\sin x}}{\sqrt{2x - 1}} + \ln 2x$

- Find  $\frac{d^2y}{dx^2}$  if  $y = 2e^{\sin x}$ .
- Evaluate:  $\lim_{x \rightarrow 2} \frac{\ln x - \ln 2}{x - 2}$

- The function  $f(x) = \ln(\cos x)$  is defined for all  $x$  in which of the following intervals?

- (a)  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$  (b)  $0 < x < \pi$  (c)  $0 \leq x \leq \pi$   
 (d)  $-\frac{\pi}{2} < x < \frac{\pi}{2}$

- A trough 4 feet long has ends which are shaped like equilateral triangles as shown. Find the volume of water in the trough when it is  $h$  feet deep.



- If  $h(x) = f(g(x))$ ,  $f(x) = \ln x$ , and  $h(x) = \ln \sqrt{x^2 + 1}$ , then find the equation of  $g$ .